An Unsteady Interactive Separation Process

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Recently, Sychev addressed the question of unsteady interactive separation moving upstream along the wall. Here, by a least-square procedure, an interaction solution is shown to exist in which the boundary-layer approximation remains uniformly valid. No additional vortex layers are introduced in the interaction region, thus avoiding the difficulties inherent in Sychev's proposal. Remarkably, according to the solution, the principal mechanics of separation is a bifurcation-free divergence of streamlines.

Introduction

UNSTEADY boundary-layer separation is one of the major fundamental, yet unresolved questions of fluid mechanics. Thus in experimental or numerical work, it remains unclear what are the distinctive features to look for in the results, or how to extrapolate to higher Reynolds numbers than may reliably be determined. The influence of a multitude of factors cannot confidently be estimated, and criteria are missing to judge results where they may mutually contradict each other.

As emphasized by Sears and Telionis,[1] "separation" is here understood to mean a physically significant departure of the boundary layer away from the wall. The location of zero wall shear is also often loosely denoted as separation, but Moore,[2] Rott,[3] and Sears[4] have all pointed out that for unsteady separation, or for steady separation from moving walls, zero wall shear does not indicate the same physical separation process as for the steady, fixed wall case. Neither are stagnation points a physical characteristic of the flow, since they depend on the choice of the coordinate system for a deformable body in arbitrary motion.

But when for sufficiently high Reynolds number the concept of a thin boundary layer can meaningfully be introduced, it becomes possible to define separation in the sense that this boundary layer is no longer in the immediate vicinity of the wall as compared to its thickness. And it is this distinction in the location of the boundary layer that will make the physically crucial difference in other flow characteristics as well: wall pressure gradient, wall shear, Kelvin-Helmholtz instability, roll-up, etc.

If a description of steady separation from a downstream moving wall can be derived, then the invariance of the flow equations under the Galileo transform may be used to map this one steady separation onto a family of unsteady ones, in all of which the separation point is in upstream motion compared to the wall. Sears and Telionis[1] argue that this family should be representative of the general case of separation points moving upstream along the wall; certainly the family should be an important example.

Thus the object becomes to study steady separation from a downstream moving wall. For this moving wall case, Moore, Rott, and Sears proposed independently that the condition of separation is no longer zero wall shear, but its generalization that the velocity component in the direction of the wall and the vorticity vanish simultaneously. More precisely, the Moore-Rott-Sears (MRS) conditions that should indicate separation are

$$u = u_{,y} = 0 \qquad \text{(steady)} \tag{1}$$

Experimental support was provided by the hot-wire measurements of Ludwig[5] of the steady separation process from a rotating, shrouded circular cylinder. For the downstream moving wall case, the MRS conditions typically denote a vanishing minimum in the velocity profiles, as sketched in Fig. 1.

A detailed description of the mechanics of the separation process has recently been proposed by Sychev.[6] Only a schematic review of his analysis can be given here.

Sychev assumes that, seen on a lengthscale comparable to the dimensions of the body, the Kirchhoff free streamline picture describes the separation process. Thus at the separation point the negligibly thin boundary layer leaves the wall to form a free vortex sheet enclosing a wake of small pressure gradient. It is well established[7] that at the separation point $x = x_S$ the complex flow velocity outside the boundary layer has a square root singularity,

$$u - iv \approx U_S - 2ki[x - x_S + iy]^{1/2} + \ldots \tag{2a}$$

The value of the coefficient k can only be found from a complete solution of the inviscid flow problem; it is furthermore dependent on the location of the separation point. On the wall immediately upstream of the separation station $x = x_S$, the square root in Eq. (2a) is imaginary, hence the vertical velocity component vanishes there and the flow is attached. Immediately downstream the square root is real so that the vertical velocity is now given as

$$v \sim 2k[x - x_S]^{1/2} \tag{2b}$$

which conforms to the displacement of a vortex sheet from the wall as described by

$$\delta_l \sim (4k/3U_S)[x - x_S]^{3/2} \tag{2c}$$

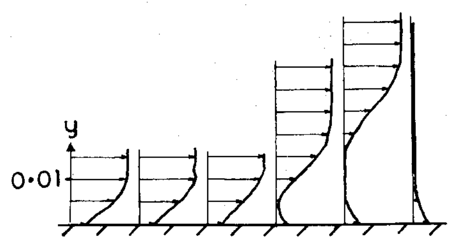

Fig. 1 Velocity profiles for separation from a downstream moving wall according to the hot-wire measurements of Ludwig.[5]

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Dedicated to William R. Sears on the occasion of his 70th birthday, with the authors' warmest wishes.

According to the Bernoulli law, the pressure in the boundary layer immediately upstream of separation has a square root singularity,

$$p=p_S-2k\rho U_S[x_S-x]^{1/2}+\ldots \tag{2d}$$

while the pressure remains approximately constant beyond the separation point.

Sychev argues that if the separation point is studied closely enough, it is seen that in a small vicinity the free streamline picture is no longer correct. Instead, in order to describe the fine details of the separation process, the small but finite boundary-layer thickness should be taken into account.

Focusing attention now first on the boundary-layer flow upstream of separation, the boundary-layer particles experience the steep adverse pressure gradient Eq. (2d) and consequently lose their kinetic energy in approaching the separation point. This gives rise to the MRS-velocity profiles sketched in Fig. 1, characterized by vanishing minimum velocity inside the boundary layer. Here the first important difference with the fixed wall case[8] becomes evident: close to a fixed wall the boundary-layer particles have vanishingly little kinetic energy. Thus those particles cannot penetrate an adverse pressure with a square root singularity (the proof follows from taking a double integral of the Von Mises equation); hence in the fixed wall case the coefficient k multiplying the singularity has to be relatively small. This effectively fixes the location of the separation point, at least for exceedingly high Reynolds numbers. But for a downstream moving wall, all particles start out with a finite amount of kinetic energy; thus the coefficient k has to be finite to reverse their motion. As a consequence, here the boundary layer rather than the inviscid flowfield is the dominating influence on the location of separation.

When the upstream boundary-layer particles lose their velocity in approaching separation, they must move laterally apart to conserve the transport of mass. As a consequence the frictional forces which they exert on one another decrease, yet on the other hand the Kirchhoff pressure gradient Eq. (2d) steepens up. Thus the viscous force rapidly loses its importance relative to the pressure gradient and the boundary-layer flow turns inviscid. Under these conditions the energy balance along the streamlines reads

$$p+{}^1\!/\!_2\rho u^2\sim p_S+{}^1\!/\!_2\rho u_S(\psi)^2 \tag{3a}$$

Since the boundary-layer flow is rotational, the total pressure is not constant, but depends on the streamline. For an MRS-separation profile as sketched in Fig. 1, u_S has to vanish at the separation streamline ψ_S. If it is further assumed that to this approximation, the curvature of the velocity profile does not vanish identically at minimum velocity, u_S must locally behave linearly:

$$u_S\sim aRe^{1/2}\,|\psi-\psi_S| \qquad \psi\to\psi_S \tag{3b}$$

The value of the coefficient a can only be found from a complete boundary-layer solution.

From this inviscid boundary-layer velocity, the penetration of the streamlines away from the wall may be estimated. Clearly the appropriate relation is the integral

$$y=\int_{\text{wall}}^{\psi}\mathrm{d}\psi'/u(x,\psi')$$

Substitution of the inviscid boundary-layer velocity equations (3) leads to an elementary integral. Thus it is seen that the boundary-layer streamlines below the separation streamline remain attached to the wall, but above the separation streamline they move away from the wall when separation is approached. More precisely, the largest contribution to the penetration away from the wall comes from the streamlines close to the separation streamline and follows as

$$\delta_I\sim Re^{-1/2}a^{-1}\ln 1/(p-p_S)+O(Re^{-1/2}) \tag{4}$$

Physically, it is the divergence of the streamlines in the immediate vicinity of the separation streamline which displaces the boundary-layer streamlines further above it away from the wall, their penetration being described by Eq. (4).

Combined with the Kirchhoff boundary-layer pressure equation (2d), the above result would imply that the boundary-layer thickness becomes logarithmically infinite at separation. But this blowup is only partly achieved, since the Kirchhoff result is only correct for a negligibly thin boundary layer. The rapidly growing boundary-layer thickness generates disturbances in the Kirchhoff outer flow velocity of order $O[Re^{-1/2}/(x-x_S)]$ and corresponding perturbations in the pressure gradient experienced by the boundary layer. It is seen that such perturbations can no longer be neglected compared to the Kirchhoff result Eq. (2d) in a small vicinity of the separation point with a typical dimension of $O(Re^{-1/3})$. In this small region the boundary layer interacts with the irrotational flow region above it by means of its displacement effect.

To describe the interactive regime, local coordinates are useful:

$$\begin{aligned}
&x-x_S=Re^{-1/3}(U_S/4ka)^{2/3}X\\
&y=Re^{-1/3}(U_S/4ka)^{2/3}Y\\
&u=U_S+Re^{-1/6}(2k^2U_S/a)^{1/3}U\\
&v=Re^{-1/6}(2k^2U_S/a)^{1/3}V\\
&p-p_S=\rho Re^{-1/6}(2k^2U_S^4/a)^{1/3}p^* \qquad y=0\\
&\delta_I=Re^{-1/2}(6a)^{-1}\ln Re+Re^{-1/2}(2a)^{-1}\delta
\end{aligned} \tag{5}$$

In terms of these local coordinates, the flow becomes independent of the values of the Reynolds number and the constants U_S, a, and k. In particular the boundary-layer displacement effect Eq. (4) becomes

$$\delta=2\ln(-1/p^*) \tag{6}$$

where both the boundary-layer pressure p^* and the governing boundary-layer thickness $\delta=\int V\mathrm{d}X$ are a priori unknown.

Since the flow above the separating boundary layer is irrotational, it can be described by a complex velocity $W=V+iU$ which is analytic with respect to the coordinate $Z=X+iY$. And if Kirchhoff free streamline results are to be correct, this complex velocity should appear to have the square root singularity (2a) for values of Z large enough that the fine details of the separation process are no longer visible. In local coordinates this amounts to

$$W\sim Z^{1/2} \qquad Z\to\infty \tag{7}$$

In Fig. 2 the boundary-layer pressure distribution for the interaction region as proposed by Sychev is sketched; it increases until it reaches a certain, still negative value and remains thereafter constant. In this second part of constant pressure, the divergence of the boundary-layer streamlines as described by Eq. (6) comes to a halt. To explain the continuing separation, which should conform to Eq. (2c), Sychev introduces in the boundary layer at the original location of minimum velocity a new region of slightly reversed flow. The upstream transport of fluid from the wake in this region of reversed flow is balanced by the entrainment of the vorticity layers enclosing the region, Fig. 2.

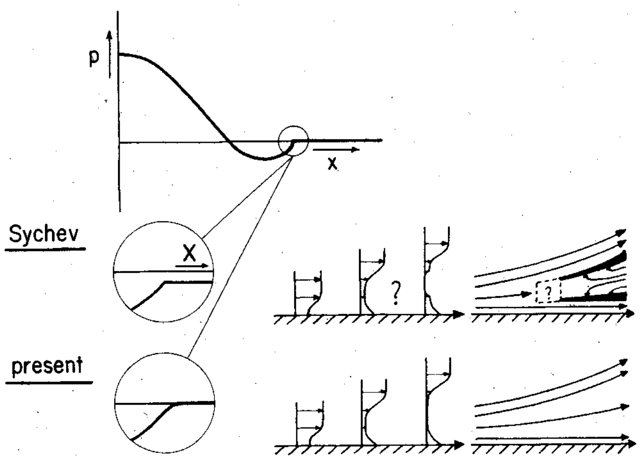

Fig. 2 Typical free streamline boundary-layer pressure distribution and corresponding interaction solutions near the separation point according to the proposals of Sychev[6] and the present.

Sychev does not give a description of the actual formation of flow reversal, but only notes that the region in which it occurs, marked with a question mark in Fig. 2, has a streamwise extend $O(Re^{-1/2})$; in this reversal region the boundary-layer approximation would no longer be valid.

Since the vortex layers leaving the reversal region are diffusing out, it would seem inevitable that there are vorticity extrema, maxima and minima, in the reversal region. Yet taking the double integral of the vorticity equation over a region enclosed by any line of constant vorticity enclosing such an extremum,

$$\nu \oint \frac{\partial \omega}{\partial n} \mathrm{d}s = 0 \tag{8}$$

hence the extremum would not decay.

Further, Ludwig's experimental velocity profiles, as sketched in Fig. 1, do not seem to suggest the two new vorticity layers introduced by Sychev. It appears surprising that these layers, with a velocity fall-off that can be estimated as $O(Re^{-1/12})$ would be lost in the experimental scatter.

The reversal region should be characterized by two special points: the first point of reversed flow should of course satisfy the MRS conditions, Eq. (1), while the dividing streamline in the downstream region of reversed flow should terminate in a stagnation point. Yet the streamlines which Inoue computed from the parabolized Navier-Stokes equations,[9,10] Fig. 3, seem to suggest that the actual formation of the separation bubble, as denoted by maximum curvature of the streamlines away from the wall, Eq. (2c), takes place well in advance of the MRS and, in particular, the stagnation point.

This paper is concerned with the possibility of doing away with the two new vortex layers introduced by Sychev. It is to be examined whether it is possible to achieve the separation by divergence of the boundary-layer streamlines alone, where the streamlines above the separation streamline closely follow the wall at the upstream end of the interaction region but separate as a free vortex sheet, Eq. (2c), at the downstream end. It means that a solution is needed to the interaction problem Eqs. (5-7) without the modifications which Sychev introduced to allow for the region of reversed flow. Such a solution would imply that the MRS and stagnation point do not come as close as $O(Re^{-1/3})$ to separation, although they may still trail vanishingly close downstream.

Solution to the Interaction Problem

According to the last section, a complex perturbation velocity W is sought which is an analytic function of the complex coordinate Z in the flow region and behaves as a

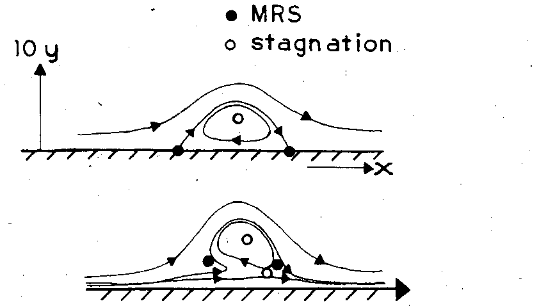

Fig. 3 Streamlines for separation from fixed and downstream moving walls according to the parabolized Navier-Stokes results of Inoue.[9] The top streamline is located rather low in the upstream boundary layer.

square root at larger values of Z. At the wall a nonlinear boundary condition to W is provided by the boundary-layer displacement effect Eq. (6). This boundary-layer result may be rewritten in the form

$$\epsilon = p^* + \exp(-\tfrac{1}{2}\delta) = 0 \tag{9}$$

where the boundary-layer thickness δ follows as the integral of the real part of W, while the pressure equals minus the imaginary part on behalf of the Bernoulli law.

Before solving the nonlinear boundary value problem for W, it is helpful to examine the behavior of possible solutions when Z is large. Since asymptotically W should behave as $Z^{1/2}$, Eq. (9) shows that the boundary-layer pressure p^* must vanish exponentially at the downstream end of the interaction region. Thus a tentative asymptotic expansion for W in powers of Z would proceed as

$$W \sim Z^{1/2} + \sum_n a_n Z^{-n/2} \tag{10}$$

where the coefficients a_n are all real and $n > -1$. Focusing next on the wall boundary condition at the upstream end of the interaction region, it is seen that the allowable values of n in the expansion are integers. In particular it does not prove possible to add powers with n not an integer, or powers with logarithmic factors, to the expansion. It is also seen that the value of the coefficient a_2 should equal -1, but the value of the coefficient a_1 cannot be found in this way. The reason is that any solution to the boundary value problem may be translated freely along the X axis; the problem does nowhere define an origin to X. It is this indeterminacy in the origin that reflects in the indeterminacy of the coefficient a_1; in this paper the origin for X was made determinate by arbitrarily choosing a_1 to vanish.

A numerical representation of the complex velocity W consistent with the obtained asymptotic expansion is

$$W = \zeta^{1/2} + A_1/\zeta - 1/\zeta^2 + \sum_{n=3}^{N} A_n/\zeta^n \qquad \zeta = [Z + iY_0]^{1/2} \tag{11}$$

Thus W is approximated by means of a finite number of unknown complex coefficients A_n. The positive constant Y_0 keeps W analytic in the flow region; a numerical value 3 was found to lead to an accurate description of W and used, even though a value 6 proved a little more accurate when the number of unknown A_n increases.

The above approximation for W has the correct square root singularity at infinity. The condition of vanishing coefficient a_1 in the asymptotic expansion for W, needed to fix the origin of X, translates to

$$\mathrm{Re}(A_1) = 0 \tag{12a}$$

In order that the error in the wall boundary condition Eq. (9) remains finite at the upstream end of the interaction region,

$$\mathrm{Im}(A_1) = -Y_0/2 \tag{12b}$$

Appropriate values for the unknowns A_n in the numerical representation of W were found from minimizing the error ϵ in the wall boundary condition Eq. (9) in the least-square sense. The minimization was done on a large number of collocation points distributed along the entire wall; the chosen distribution was

$$X_k = L\left[\frac{k}{K}-\frac{1}{2}\right]\Big/\left[2\frac{k}{K}\left(1-\frac{k}{K}\right)\right]^2 + X_0 \qquad k=1,2,\ldots,K-2$$

$$X_{K-1} = -10^4 L \tag{13}$$

The constant L denotes a characteristic spread of the collocation points and X_0 the location of maximum point density. Values $L=6$ and $X_0=-3$ were used in the presented results, but accuracy depended little on the precise values. The number $K-1$ of collocation points was in general chosen as 20 times the number of unknown A_n, but doubling or halving this did not significantly alter the results.

To solve the least-square equations, if for convenience the unknown real and imaginary parts of the A_n are denoted by U_1, U_2, ..., the solution was found iteratively according to

$$\sum_{k,r'} \epsilon_{k,r}\epsilon_{k,r'}\Delta U_{r'} + \epsilon_k \epsilon_{k,r} = 0 \qquad r=1,2,\ldots \tag{14}$$

The solution converged in general very fast and no dependence of the solution on the initial guess was observed; usually as initial guess the A_n were set to zero. The chosen representation (11) for W makes for a numerically efficient evaluation of the ϵ_k and $\epsilon_{k,r}$ and computational resources on the IBM 370 computer were negligible. Double precision proves necessary however.

Computational overflow in the evaluation of the residues first occurred at 11 unknown coefficients A_n, but results for nine unknown A_n were already judged sufficiently accurate.

Results

The numerical results for the flow in the interaction region showed that indeed the wall boundary condition may be satisfied to high accuracy, as shown in Table 1. Moreover, the resulting boundary-layer pressure distribution, partly shown in Fig. 4, appeared to converge within the error tolerance ϵ in the boundary condition.

Thus the boundary value problem, which may be viewed as a nonlinear singular integral equation for the wall pressure distribution, does appear to have a meaningful solution.

Similarly the resulting boundary-layer pressure gradient and displacement thickness appear meaningful; this is of course made possible by the appropriate numerical representation Eq. (11) of the solution. The maximum error in the displacement thickness occurs at fairly large and rapidly increasing values.

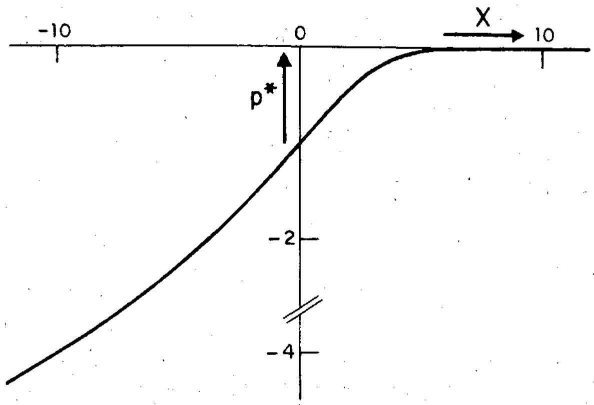

Fig. 4 Present boundary-layer pressure distribution in the vicinity of separation. A family of other solutions is obtained by shifting this solution and Fig. 5 along the X axis.

In the results, the origin of X was determined rather arbitrarily, by demanding that the second coefficient a_1 in the asymptotic expansion Eq. (10) of the solution vanishes. But in fact any translation of the solutions Figs. 4 and 5 along the X axis is also a valid solution to the boundary value problem.

A slightly different numerical approach is to choose the origin of X by maximizing the numerical accuracy; in the terminology of the last section, this means that $\mathrm{Re}(A_1)$ is treated as a variable in the least-square minimization, rather than that Eq. (12a) is imposed. Thus the wall boundary condition may be satisfied to still higher accuracy, Table 1. The solutions may be shifted back along the X axis to coincide with the one of Fig. 4.

Table 2 lists values for the coefficients A_n in the numerical representation Eq. (11) of the solution. It is seen that the values do not converge with increasing number of coefficients. Yet according to Table 1 those differences do not reflect in differences in the solutions for the boundary-layer pressure distribution and displacement thickness. This must of course mean that the representing functions in the representation Eq. (11) of the solution become nearly linearly dependent. But no attempt was made to reduce the number of unknowns by orthogonalization of the system of functions.

No description for the flow farther downstream of the interaction region was derived. It may be observed, though, that the eventual entrainment of the separated upper and attached lower part of the boundary layer should be given by

$$v = -u\int_{\mathrm{wall}} \nu u_{,yy}/u^2 \mathrm{d}y \sim -\ a\nu Re\, Re^{-1/2} \tag{15}$$

Thus a finite distance downstream of separation, the amount of entrained fluid ought to be comparable to the total amount of fluid in the upstream attached boundary layer. The origin of this entrained fluid would be difficult to explain if there would not be flow reversal, indicated by MRS and stagnation points, trailing vanishingly close to the separation point.

From another point of view, leaving the interaction region at the downstream end the boundary-layer pressure gradient

Table 1 Numerical solution of the interaction problem for increasing number N of terms in its representation, Eq. (11)[a]

	$a_1=0$				$a_1 \neq 0$	
N	ϵ	$p^{*\prime}_{\max}$	ϵ'	ϵ''	ϵ	$p^{*\prime}_{\max}$
4	0.06835	0.28319	0.00932	0.414	0.02020	0.29155
7	0.00703	0.29002	0.00125	0.039	0.00073	0.28992
10	0.00091	0.29008	0.00022	0.013	0.00004	0.28994
11	0.00053	0.28990	0	0	0.00002	0.28994

[a] The $N=11$ solution is partly shown in Figs. 4 and 5 and was used as a reference. ϵ denotes the maximum error in the wall boundary condition Eq. (9), $p^{*\prime}_{\max}$ the maximum boundary-layer pressure gradient, ϵ' the maximum deviation in boundary-layer pressure gradient from the reference solution, and ϵ'' the maximum deviation in displacement thickness δ. See text for the significance of a_1.

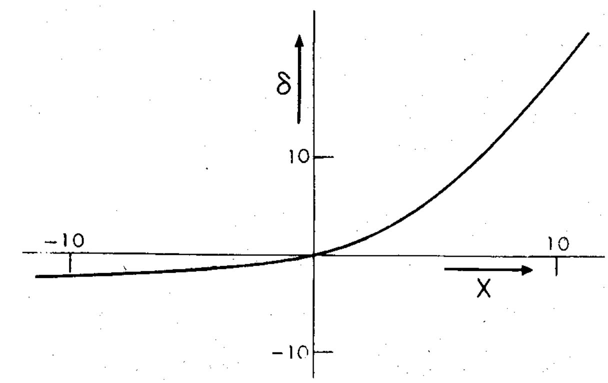

Fig. 5 The variable part δ of the boundary-layer displacement thickness in the vicinity of separation.

Table 2 Coefficients $A_3, A_4, \ldots, A_N$ in the numerical representation Eq. (11) of the solution, for various N. They correspond to $Y_0 = 3$ and $A_1 = -1.5i$

N				
4	0.39972	$-0.01300i$	-0.22068	$-0.71934i$
7	1.18767	$+0.00467i$	0.37268	$-2.67841i$
	1.16082	$-2.78915i$	-9.20047	$-7.83052i$
	-3.80093	$+9.22937i$		
11	1.29371	$+0.00004i$	0.00311	$-5.28447i$
	-20.15783	$+2.48709i$	24.32671	$+92.05630i$
	313.29644	$-141.21820i$	-502.58923	$-626.30098i$
	-605.03668	$+980.94670i$	918.26907	$+124.75534i$
	-107.79358	$-301.53476i$		

decays exponentially, Eq. (9), Fig. 4. Thus the pressure forces lose their dominance and the originally inviscid boundary layer should again turn viscous.

Thus, plausibly, behind the interaction region a regime of viscous flow is found in which actual flow reversal is achieved. Since the boundary-layer flow in the interaction region is approximately symmetric about the separation streamline, Eq. (3b), plausibly the MRS point would to first approximation also be a stagnation point; this would agree closely with the original MRS idea.[3] Similar to the fixed wall case, the boundary-layer equation does allow a formal solution which passes the MRS point regularly. True, for prescribed pressure gradient the boundary-layer solution may feature a singularity of the form Eq. (4), as discovered independently in Lagrangian analysis by Van Dommelen and Shen.[11-13] However, for the boundary-layer flow in the reversal region, the displacement thickness rather than the pressure gradient should be prescribed; the singularity of Van Dommelen and Shen is inconsistent with a prescribed displacement thickness.

Conclusion

Unsteady separation is a major unsettled question of fluid mechanics. By modifying recent ideas of Sychev,[6] in this paper possible mechanics for the separation process were derived when the separation point is in upstream motion compared to the wall. The framework was a Kirchhoff free streamline theory, which may render the result inapplicable to the first separated stages after the onset of separation.[13]

Seen in a coordinate system moving along with the separation point, during the separation process the minimum velocity in the boundary layer falls off sharply under the acting large adverse pressure gradient. To preserve transport of mass, the boundary-layer streamlines at minimum velocity expand rapidly, displacing the streamlines above minimum velocity away from the wall; eventually those upper boundary-layer streamlines lift off from the wall in the form of a free vortex sheet.

Thus the principle mechanics of the separation process is here a rapid divergence of the boundary-layer streamlines near minimum velocity. Qualitatively those features appear to agree well enough with the experimental results of Ludwig, Fig. 1, and the numerical ones of Inoue, Fig. 3.

Besides the flat minimum in the velocity profile, the most distinctive signs of this separation process are maximum boundary-layer pressure gradient and upward streamline curvature. In Ludwig's experiments, this large pressure gradient, and hence streamline curvature, Eqs. (2), was deliberately reduced by shrouding of his body. No pressure data are given by Inoue.

No solution was given for the already separated flow immediately downstream. Yet, according to the previous section, it does seem inevitable that trailing vanishingly close downstream of separation, MRS and stagnation points ought to be located. This does not conform with the results of Inoue, Fig. 3, but since here the separation bubble is very slender, possibly the divergence of the boundary-layer streamlines does not go far enough for the viscous force to come back into the picture. The experimental results of Ludwig provide little guidance here, since they describe only the absolute magnitude of the velocity.

According to the solution, the resolution needed to resolve for the separation process is of the order $O(Re^{-1/3})$. This may provide some guidance for numerical analysis; for the example of the circular cylinder, common practice is to keep streamwise mesh spacing at from 15 to 20 deg, regardless of the Reynolds number.

The developed least-square procedure may of course easily be incorporated in a least-square determination of complete separating flowfields at high Reynolds number. At least this part of such a computation would require little computational resources.

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